Solutions 3

1

- $y_0 = 0$: Interpolation along x

x	z	
0	0	
1	1	1

$z := p(x,y)$

$$\Longrightarrow p(x, y_0 = 0) = 0\,\pi_0 + 1\,\pi_1(x) = 1(x - x_0) = x$$

- $y_0 = 1$: Interpolation along x

x	z	
0	1	
1	0.5	-0.5

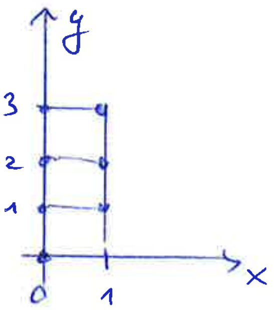
$$\Longrightarrow p(x, y_0 = 1) = 1\,\pi_0 - 0.5\,\pi_1(x) = 1 - 0.5x$$

- Interpolation along the variable y:

y	z	
0	x	(!)
1	$1 - 0.5x$	$1 - 1.5x$

$$\Longrightarrow p(x,y) = x\,\pi_0 + (1 - 1.5x)\,\pi_1(y) = x + (1 - 1.5x)\,y = x + y - 1.5xy$$

2



$p(x,y)$: Bi-cubic, 16 free coefficients.

a) Osculation along x

Prerequisites: $p(0,0)$; $p(1,0)$
$p(0,1)$; $p(1,1)$
$p(0,2)$; $p(1,2)$
$p(0,3)$; $p(1,3)$
$\frac{\partial p}{\partial x}(0,0)$; $\frac{\partial p}{\partial x}(1,0)$
$\vdots$ $\vdots$
$\frac{\partial p}{\partial x}(0,3)$; $\frac{\partial p}{\partial x}(1,3)$

16 "conditions"

Collocation along y

b) x: $\{1;\ x;\ x^2;\ x^2(x-1)\} =: A$

y: $\{1;\ y;\ y(y-1);\ y(y-1)(y-2)\} =: B$

(Sol. 3 cont.)

2 (cont.) $A \times B$ (Product) $= \{1;\ x,\ x^2,\ x^2(x-1);\ y,\ yx,\ yx^2;\ yx^2(x-1);\ y(y-1),\ y(y-1)x,\ y(y-1)x^2,\ y(y-1)x^2(x-1),\ y(y-1)(y-2),\ y(y-1)(y-2)x,\ y(y-1)(y-2)x^2,\ y(y-1)(y-2)x^2(x-1)\}$

This bivariate basis consists of 16 functions.

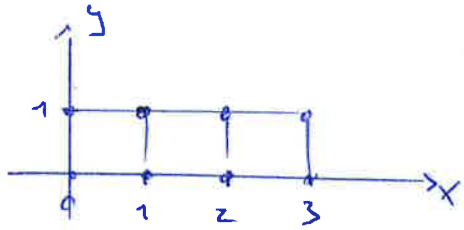
3 x: $\{\pi_0;\ \pi_1(x)\} = \{1,\ (x-x_0)\} = A$

y: $\{\pi_0;\ \pi_1(y)\} = \{1,\ (y-y_0)\} = B$

z: $\{\pi_0;\ \pi_1(z)\} = \{1,\ (z-z_0)\} = C$

$A \times B \times C = \{1,\ (x-x_0),\ (y-y_0),\ (z-z_0),\ \ldots\ (x-x_0)(y-y_0)(z-z_0)\}$ (8 functions)

$= \{\pi_i(x)\,\pi_j(y)\,\pi_k(z)\ |\ i,j,k \in \{0,1\}\}$

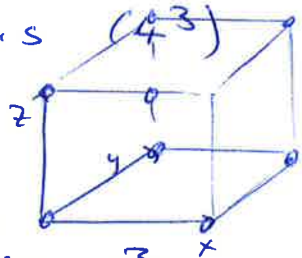
4 

- x Cubic Interpolation ($y_0 = 0\ /\ y_0 = 1$)
  Basis: $\{1,\ x,\ x(x-1),\ x(x-1)(x-2)\}$
- y Linear Interpolation ($x_0 = 0\ /\ 1\ /\ 2\ /\ 3$)
  Basis $\{1,\ y\}$

Product $= \{1,\ x,\ x(x-1),\ x(x-1)(x-2),\ y,\ yx,\ yx(x-1),\ yx(x-1)(x-2)\}$

5 a) 8 because 8 basis functions (cf. 3)

b) 64 " 64 basis functions (4^3)



6 Collocation:

8 prerequisites: Values $p(x_0, y_0, z_0)$ for $(x_0, y_0, z_0) \in \{0;1\}^3$

Osculation: . . .

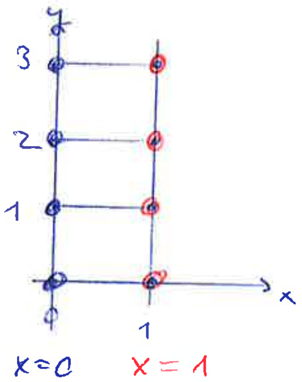
(Sol. 3 cont.)

6 (cont.)

- $\frac{\partial p(x_0, y_0, z_0)}{\partial x}$; $\frac{\partial p}{\partial y}(\ldots)$; $\frac{\partial p}{\partial z}(\ldots)$
  (8 prereq.) ; (8 prereq.) ; (8 prereq.)
- $\frac{\partial^2 p}{\partial x \partial y}(x_0, y_0, z_0)$; $\frac{\partial^2 p(\ldots)}{\partial x \partial z}$; $\frac{\partial^2 p(\ldots)}{\partial y \partial z}$
  (8) (8) (8)
- $\frac{\partial^3 p(x_0, y_0, z_0)}{\partial x \partial y \partial z}$ $\left((x_0, y_0, z_0) \in \{0,1\}^3\right)$

Totally: 64 prereq. compared to $4^3 = 64$ basis functions (cf. 5b)

2 (rev.) Alternative solutions to 2



a_1) Osculation along x

Prerequisites: $p(x,y)$ for $x \in \{0,1\}$ and $y \in \{0,..,3\}$

Derivatives: $\frac{\partial p}{\partial x}(0,0)$; $\frac{\partial^2 p}{\partial x^2}(0,0)$

$\frac{\partial p}{\partial x}(0,1)$; $\frac{\partial^2 p}{\partial x^2}(0,1)$

$\frac{\partial p}{\partial x}(0,2)$; $\frac{\partial^2 p}{\partial x^2}(0,2)$

$\frac{\partial p}{\partial x}(0,3)$; $\frac{\partial^2 p}{\partial x^2}(0,3)$

b_1) ⓧ: $\{1, x, x^2, x^3\} =: A$ ($x=0$ is "repeated" twice)

ⓨ: $\{1, y, y(y-1); y(y-1)(y-2)\} =: B$

a_2) Derivations: $\frac{\partial p}{\partial x}(1,0)$; $\frac{\partial^2 p}{\partial x^2}(1,0)$

$(1,1)$; $(1,1)$

$(1,2)$; $(1,2)$

$(1,3)$; $(1,3)$

b_2) ⓧ: $\{1; x; x(x-1); x(x-1)^2\}$ ($x=1$ is "repeated")