Exercises 12: Numeric Ordinary Differential Equations II

The problems are solvable with a computer, normally. There are exceptions.
The symbol **|** means „or“, the symbol ***** „optional“, the symbol ****** „optional and advanced“ and the symbol © means that a computer is required or helpful.

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1. **a)** © Solve the initial value problem $y' = x\,y^{1/3} \quad y(1) = 1$ numerically by the classical Runge-Kutta method of order p = 4 and fixed step-size h = 0.1 for the x-values 1.1 und 1.2 (two steps). All final (!) results should be rounded to the 10th digit.

b) © Compute the local error (slope) $\tau_h(x_n)$ and from this $h \cdot \tau_h(x_n)$ as well as the global error for the two steps in a).

Hints: The exact solution of the equation is $y = \left(\frac{x^2+2}{3}\right)^{3/2}$. Compare your results those of Problem 1 from Exercises 11.

2. What is the crucial advantage of the Runge-Kutta method of order 4, i.e. RK4, over the Taylor method of order 4.

3. © Compute the coordinates of the $(t, \overset{4}{\cancel{\square}})$ point after the first step rounded to the 10th digit in Example 1.5 / Figure 1.4 on page 11/12 (script).

Additionally, compute the local error (of slope) $\tau_h(t_n)$ and from this the increment $h \cdot \tau_h(t_n)$ as well as the global error. Contrast the answers to those of Exercises 11, Problem 2.

Hints: A reference value for the exact $\overset{\varphi}{\square}$ is 0.2837468640107449 (rounded to the 16th digit). The examples refer to the differential equation $\varphi' = c\left(1 - \varepsilon\cos\varphi\right)^2 \quad \varphi(0) = 0$ with c = 1 and $\overset{\varepsilon}{\square}$ = 0.25 (angle-velocity-relation of an elliptical gravitational orbit).

4. Describe the advantage of a FSAL-Runge-Kutta scheme (First-Same-As-Last). What are the savings?

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5. ** Derive the general equations for an explicit Runge-Kutta method of order p = 3 and verify the Butcher tableau below as (a correct) solution. The computations here are rather lengthy (from Schaum's Outline of Numerical Analysis, Chapter 19, P19.9), cf. the solution files.

$$\begin{array}{c|ccc} 0 & & & \\ 1/2 & 1/2 & & \\ 1 & -1 & 2 & \\ \hline & 1/6 & 4/6 & 1/6 \end{array}$$