Exercises 2: Polynominterpolation III

The problems are solvable without a computer, normally. There are rare exceptions.
The symbol **|** means „or", the symbol * „optional", the symbol ** „optional and advanced" and the symbol © means that a computer is required or helpful.

1. **[Schaum 10.3]** The points (0,0) and (4,2) are to be connected by a low degree polynomial $p(x)$. The tangents drawn in the figure are horizontal. Compute the connecting polynomial p and indicate its x-range.

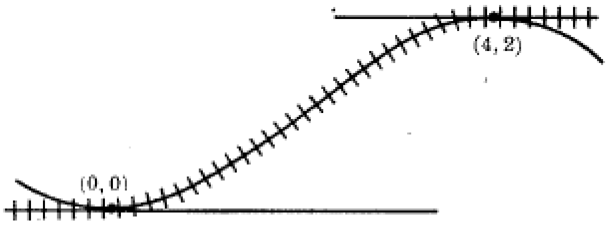

Fig. 10-1

Hint: What is the expected low degree of p ?

2. * **[Schaum 10.6]** Write down a symbolic expression representing a low-degree polynomial interpolating the symbolic data:

x_0	y_0	y_0'	y_0''
x_1	y_1	y_1'	y_1''

3. **[Schaum 10.10]** Compute an interpolating low-degree polynomial for the data:

x_k	y_k	y_k'	y_k''
0	0	0	0
1	1	1	0

Additionally, bound the interpolating error assuming that $\max_{0\le x\le 1}\left|y^{(6)}(x)\right| \le M = 10$. Give an algebraic and a numerical bound, respectively.

4. **[Schaum 10.12]** Compute two fourth-degree (!) polynomials, $p_1(x)$ and $p_2(x)$, meeting the constraints below:

$$p_1(0) = p_1'(0) = p_1''(0) = 0$$

$p_2(4)=2,\ p_2'(4)=p_2''(4)=0$ and $p_1''(2)=0,\ p_2''(2)=0$.

Moreover, the two polynomials should meet smoothly at the point (2,1) without a crinkle (with a common tangential line).

Hint: Use a symbol, e.g. a, for the unknown derivative $p_1'(2)=p_2'(2)$. Finally, note that the degree of both of the polynomials is 4.

5. **[Schaum 10.15-10.16]** Compute low-degree polynomials interpolating the data sets below:

a)

x_k	y_k	y'_k
0	1	0
1	0	—
2	9	24

b)

x_k	y_k	y'_k	y''_k
0	1	−1	0
1	2	7	—

© Additionally, bound the interpolating errors assuming that $\left|y^{(5)}(x)\right| \le M = 100$ in the corresponding x-range. Give algebraic and a numerical bounds, respectively.

6. Represent the divided differences below as higher derivatives:

a) $y(\underbrace{x_0, x_1, \ldots, x_n, x}_{n+2}) \quad x \neq x_k \quad (k=0,\cdots,n)$

b) $y(\underbrace{x_n, x_1, x, x_2 \ldots, x_{n-1}}_{n+1}) \quad x \neq x_k \quad (k=1,\cdots,n)$

c) $y(\underbrace{x_0, x_0, \ldots, x_0}_{n+1})$

7. © The model function $y(x)=\cos(\frac{1}{2}\pi\, x)$ has to be interpolated within the arguments x = 0;1 by a low degree polynomial $p(x)$ including the values of the first derivatives at the boundaries x = 0;1 (osculation problem).
Estimate the absolute maximum error $\max\limits_{x\in[0,1]}\left|y(x)-p(x)\right|$ by a simple expression and check your estimation at the position x = ½ by comparing with the exact interpolation error.

8. What are advantages versus disadvantages when using Chebyshev arguments for polynomial interpolation?