Exercises 3: Multivariate Polynomial Interpolation

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1. A bivariate function $p(x, y)$ has the values listed below in the corners of the regular square grid {0,1} x {0,1} :

$$p(0,0) = 0,\ p(0,1) = 1,\ p(1,0) = 1,\ p(1,1) = 0.5$$

Compute the bilinear interpolation $p(x, y)$ by hand.

2. {0,1} x {0,1,2,3} is a regular bivariate grid and would be suitable for a bi-cubic polynomial interpolation $p(x, y)$. It is not the task to compute a formula for $p(x, y)$.

a) Find a set of simple requirements (values, derivatives) for $p(x, y)$ at the points of the grid that guarantee a solution for the interpolating problem.

b) Write down a set of bivariate Newton basis polynomials compatible with your answer in a).

3. List the 3-fold (tri-variate) tensor product of the univariate Newton basis $\{\pi_0, \pi_1\}$ in the variables $\{x, y, z\}$.

4. Write down a suitable basis of bivariate Newton basis polynomials for a collocation on the regular grid {0,1,2,3} x {0,1}?

5. How many conditions are generally required

a) for a tri-linear …

b) for a tri-cubic polynomial interpolation

on the regular cube grid {0,1} x {0,1} x {0,1} = $\{0,1\}^3$?

6. Write down a set of simple sufficient requirements (values, derivatives) for a tri-cubic interpolation $p(x, y, z)$ at the points of the regular cube grid {0,1} x {0,1} x {0,1} = $\{0,1\}^3$.
