Exercises 5: Spline Interpolation II (Bernstein Bézier Splines)

The symbol **|** means „or“, the symbol * „optional“, the symbol ** „optional and advanced“ and the symbol © means that a computer is required or helpful.

1. Script Spline Interpolation, Example 2.3:

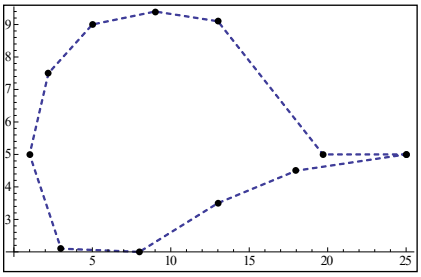
a) Since a cubic natural spline interpolation for a given data set is unique the spline interpolation of Example 2.3 must be the same as in Exercises 4, 1a). Show this for the first patch [0, π/3].

b) Estimate the maximum error $|\, y(x) - S(x) \,|$ of the spline interpolation in Example 2.3.

2. © The points Q_0 to $Q_{11} = Q_0$ (counterclockwise) outline the cross-section of an airfoil body. At Q_0 (the point to the most right) there is a sharp cusp (!).

The figure shows a piecewise linear spline interpolation.

The problems in this exercise deal with a composed cubic C^2 Bézier spline interpolating the data given below (apart from Q_0 where the connection only is continuous C^1).



	Q0	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11
x	25	19.7	13	9	5	2.2	1	3	8	13	18	25
y	5	5	9.1	9.4	9	7.5	5	2.1	2	3.5	4.5	5

a) Write down linear equations for the 4 control points going along with the data points $Q_{11} = Q_0$ and Q_1 as well as for the 2 control points "before" Q_0 .

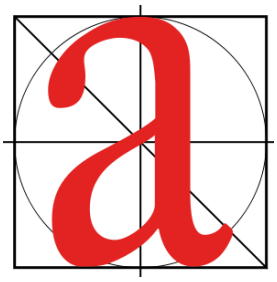
b) How many control points must be provided totally ?

c) How many equations can be formulated using the informations in the introductory text above?

d) © Compute and plot the composed C^2 Bézier spline and examine the situation at the cusp (Q_0).

Hints: S. Example 2.3. Denote the control points going along with Q_j (j = 0, 1, …, 10) by P_{ij} (i=1,2; j = 0, 1, …, 10). The cusp at Q_0 implies two singular equations (!).

3. © These problems concerning font design are perfectly analogous to Examples 2.1, 2.2 in the script spline interpolation.



An extract from a .svg file of the Adobe a (figure on the left) has the lines:

```
M 183.171,159.905
c -19.081,16.712-71.022,29.131-71.022,80.636
c 0,22.975,11.665,34.132,27.556,34.132
c 15.372,0,43.467-21.71,43.467-38.475
```

a) Compute all Bézier splines involved with the code extract above and plot the composed Bézier spline.

b) Check the smoothness condition C^1 of the composed spline at the two connection points.

Hints: Beside the script you can check the *Mathematica* notebook `Case_FontDesignBezier1.nb`.