Exercises 8: Linear Least-Squares Approximation (multivariate discrete data)

The problems are solvable without a computer, normally. There are exceptions.
The symbol **|** means „or“, the symbol ***** „optional“, the symbol ****** „optional and advanced“ and the symbol © means that a computer is required or helpful.

1. Normally, a set of four 3d-points (x, y, z) is not contained in one single plane. But generally there is a plane coming close to the 3d-points in the sense of least-squares approximation.

The (x, y, z)-data in this problem is: $A = (1,0,0),\ B = (0,1,0),\ C = (0,2,-1),\ D = (1,3,1)$

a) Give a reasonable set of basis functions. *Hint*: A plane has total degree 1 (complete basis).
b) Write down the design matrix according to a) and the normal equations.
c) © Solve the system of normal equations and give a functional formula for the approximating plane.
d) * © Plot the data combined with the approximating plane.

2. © This problem refers to Example 1.7A in the least-squares approximation script and the *Mathematica* file `Case_3DHandLeastSquares.nb`.

a) For the complete basis of total degree 3 recover the computations for the diagonal matrix in the singular value decomposition (only 10 rows) and the least-squares approximation written down in the script example 1.7A.
b) Compute the ratio of the absolutely maximum singular value and the absolutely minimum singular value of the design matrix. By the way: The square of this ratio is the condition number of the normal matrix.

3. © This problem continues Problem 2 above. Redo the computations in problem 2 for the normalized complete basis of total degree 3, i.e. x is substituted by $\frac{x-\mu_x}{\sigma_x}$ and y is substituted by $\frac{y-\mu_y}{\sigma_y}$. The symbols μ and σ, respectively, denote the mean and standard deviation of the x and y data, respectively. Compare the results with those of problem 2.